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1 SEM TDC GEMT (CBCS) GE 1 (A/B/C)

2021

(Held in January/February, 2022)

MATHEMATICS

(Generic Elective)

Paper : GE–1

The figures in the margin indicate full marks for the questions

Paper : GE–1 (A)

(Differential Calculus)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. (a) Define limit of a function at $+\infty$. 1

(b) Write the type of discontinuity if

$$\lim_{x\to a-0} f(x) \neq \lim_{x\to a+0} f(x)$$ 1

(c) Find a positive number N satisfying $|f(x)-L|<\varepsilon$ if $x>N$ where

$$\lim_{x\to\infty} \frac{x}{1+x} = 1, \ \varepsilon = 0\cdot 001$$ 3

Or

Find $\lim_{x \to \infty} \frac{5x^2 - 4x}{2x^2 + 3}$.

(d) Show that $\lim_{x \to 2} \frac{2x^2 - 8}{x - 2} = 8$. 2

(e) Show that $f(x) = |x|$ is continuous everywhere. 3

Or

A function f is defined by

$$f(x) = \begin{cases} -x^2, & \text{if } x \le 0 \\ 5x - 4, & \text{if } 0 < x \end{cases}$$

Examine the continuity of f at $x = 0$ and write the type of discontinuity, if any.

2. (a) Discuss the derivability of the function

$$f(x) = \begin{cases} 2, & \text{if } x \le 1 \\ x, & \text{if } x > 1 \end{cases}$$

at $x = 1$. 2

(b) Prove that if a function f is differentiable at $x = c$, then f is continuous at c. 3

(c) Write a sufficient condition for equality of f_{xy} and f_{yx}. 1

(d) If $y = \sin(ax + b)$, then find y_n. 2

(e) Examine the equality of f_{xy} and f_{yx} for the function $f(x, y) = x^3 y + e^{xy^2}$. 3

3. (a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, then show that

$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x + y + z}$$ 4

Or

If $u = \tan^{-1} \frac{x^3 + y^3}{x - y}$, then show that

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \sin 2u$$

(b) State and prove Leibnitz's theorem. 5

Or

If $y = e^{a \sin^{-1} x}$, then show that

$$(1 - x^2) y_{n+2} - (2n + 1) x y_{n+1} - (n^2 + a^2) y_n = 0$$

4. (a) Find the interval where the curve $y = x^3$ is concave up. 1

(b) Find the slope of the tangent to the curve of parametric equations

$$x = \cos t,\ y = \sin t \text{ at } t = \pi/6$$ 2

(c) Find where the tangent is parallel to the x-axis for the curve

$$y = x^3 - 3x^2 - 9x + 15$$ 3

Or

Find the equation of the normal to the curve $x^2 - xy + y^2 = 7$ at (−1, 2).

(d) Find the acceleration of the particle whose position $P(x, y)$ is given by the equations $x = t^3$, $y = t^2$, $-\infty < t < \infty$. 4

Or

Identify the symmetry of the curve and then draw the graph of the equation $r = 4 - 4\cos\theta$.

5. (a) Draw the graph of $y = x^4 - 4x^3 + 10$. Identify the inflection point, if any. 4+1=5

Or

Find the asymptotes of the curve

$$f(x) = \frac{x^3 - 1}{x^2 - 1}$$ 5

(b) Find the curvature for any curve

$$\vec{r}(t) = (a\cos t)\hat{i} + (a\sin t)\hat{j} + bt\hat{k},\quad a, b > 0$$ 5

Or

Find the radius of curvature at the point 'θ' on the curve

$$x = a(\theta + \sin\theta),\ y = a(1 - \cos\theta)$$

6. (a) State Lagrange's mean value theorem. 1

(b) State Taylor theorem with Cauchy's form of remainder. 2

(c) Find the Cauchy's remainder after n terms in the expansion of $\log(1 + x)$. 2

(d) Find the value of c in the mean value theorem $f(b) - f(a) = (b - a)f'(c)$ if $f(x) = Ax^2 + Bx + C$ in (a, b). 3

7. (a) State and prove Rolle's theorem. 5

(b) Examine the conditions of Rolle's theorem in the function

$$f(x) = (x-a)^m (x-b)^n$$

m, n are positive integers and $x \in [a, b]$. 2

(c) Find the Taylor series of order n generated by $f(x) = \sin x$ at $x = 0$. 4

8. (a) Expand $\cos x$ in powers of x in an infinite series using Maclaurin's series. 5

Or

Find the extreme values of the function

$$f(x) = x^5 - 5x^4 + 5x^3 - 1$$

(b) Evaluate (any *two*) : 3×2=6

(i) $\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x^2 \sin x}$

(ii) $\lim_{x \to 0} (\cos x)^{1/x}$

(iii) $\lim_{x \to \pi/2} (\sec x - \tan x)$

(iv) $\lim_{x \to 0} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$

Paper : GE–1 (B)

(Object-Oriented Programming in C++)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

1. Answer the following questions (any *ten*) : 1×10=10

   (a) What is class? Give an example.

   (b) What is encapsulation?

   (c) What is inheritance?

   (d) Define header file.

   (e) What is function?

   (f) What do you mean by compiler?

   (g) What is array?

   (h) Write the full form of iostream.h.

   (i) Define data member.

   (j) What is pointer?

   (k) What do you mean by operating system?

2. Answer the following questions (any *three*) : 2×3=6

   (a) What are constructors and destructors?

   (b) What is the difference between while loop and do while loop?

   (c) What is dynamic binding? Define message passing.

   (d) Define friend function.

   (e) State two differences between break and continue.

3. Answer the following questions (any *three*) : 4×3=12

   (a) How do the invocation constructors differ in derivation of class and nesting of class?

   (b) Write short notes on prefix and postfix increment and decrement operators with example.

   (c) What is file pointer? Define function prototyping.

   (d) Discuss dynamic memory allocation in detail.

   (e) Write the characteristics of object-oriented programming.

4. Answer the following questions (any *two*) : 7×2=14

(a) Explain inline and virtual functions with suitable example.

(b) What are call by value and call by reference? Explain with example.

(c) What do you mean by inheritance? Explain different types of inheritance with example.

5. Write C++ program of the following (any *three*) : 6×3=18

(a) To overload an operator

(b) To keep a count of created objects using static members

(c) To display Fibonacci series

(d) To store information of a student in a structure

Paper : GE–1 (C)

(Finite Element Methods)

Full Marks : 80

Pass Marks : 32

Time : 3 hours

1. (a) What do you mean by a finite element? 1

(b) State the heat equation as a prototype of a parabolic equation. 4

Or

State maximum principle and find its expression in maximal point.

(c) What are the different types of partial differential equations and their fields in applications? 4

(d) Illustrate the process of discretization in two-dimensional domain with a suitable example. 5

2. (a) Define a discretely connected space. Also state discrete maximum principle. 3

Or

Define a weak derivative. Find an expression for weak derivative.

(b) State the Lax-Milgram theorem and find its derivation. 3

Or

Write down the differences between finite difference methods and finite element methods.

3\. (a) Describe briefly about Ritz-Galerkin method. 3

Or

State the properties for a triangular or quadrilateral element.

(b) Using Green's theorem, find an expression for Euler's equation. 4

Or

Shape functions play a vital role in finite element methods. Justify.

(c) Find an expression for triangular elements with complete polynomials. 5

Or

Define affine families of rectangular elements with suitable diagrams.

4\. (a) Find an expression for a linear element in finite element method. 2

(b) What are the roles of natural coordinates in the process of formulation of a linear Lagrange polynomial? 3

(c) Define a rectangular element. Find an expression for linear Lagrange polynomial in case of rectangular element. 4

(d) Discuss briefly with an example about the element assemblage in finite element method. 3

Or

What is interpolating function in finite element method? Find an expression for interpolating function in one-dimensional domain.

5\. (a) Define Sobolev space in finite element method. 2

(b) Find an expression for triangular elements and hence find its stiffness equation. 3

Or

State the refinement rules in the process of triangularization.

(c) Discuss about the effects on the choice of a grid in forming variational problems. 3

(d) Write down the importance of sparse matrix in the process of element assemblage with an example. 4

Or

Solve the boundary value problem

$\nabla^2 u = 1$, on $|x| \le 1$, $|y| \le 1$

$u = 0$

on the boundary, using Galerkin method.

6. *(a)* Give an example of triangular element with a common node. 1

(b) Solve the boundary value problem

$\nabla^2 u = 0$ on $0 \le x \le 1$, $0 \le y \le 1$

under Dirichlet's boundary conditions. 5

(c) Find a one-parameter Galerkin solution of the boundary value problem

$\nabla^2 u = x^2 - 1$, $|x| \le 1$, $|y| \le ½$ and $u = 0$

on boundary. 6

Or

Write an algorithm to generate the stiffness equation in the formation of a variational problem.

7. *(a)* Find an expression for cubic hermite polynomial in constructing linear boundary value problem. 4

(b) Describe the variational method for a definite integral

$$J[u] = \int_a^b F(x, u, u')\,dx$$ 4

Or

Find the Euler equation for a variational problem with a suitable shape function.

(c) Derive the difference scheme for the boundary value problem

$u'' - ku = 0$; $u(0) = 1$, $u(1) = 0$

and $k > 0$ is assumed constant. 4

Or

Using finite element method, solve the boundary value problem :

$\nabla^2 u = -1$, $|x| \le 1$, $|y| \le 1$

$u = 0$, $|x| = 1$, $|y| = 1$ with $h = ½$

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