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3 SEM TDC MTMH (CBCS) C 7

2022

(Nov/Dec)

MATHEMATICS

(Core)

Paper : C-7

(PDE and Systems of ODE)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

*The figures in the margin indicate full marks
for the questions*

1. (a) Find the degree of the equation

$$x \frac{\partial^2 z}{\partial x^2} + y \left(\frac{\partial z}{\partial y} \right)^{1/3} + Kz = 0 \quad 1$$

- (b) Define linear partial differential equation. 1

- (c) Write the general form of Lagrange's equation. 1

- (d) Form the PDE by eliminating the arbitrary functions f and ϕ from 5

$$z = yf(x) + x\phi(y)$$

Or

Solve :

$$(x^2 - yz)p + (y^2 - zx)q = z^2 - xy$$

- (e) Find the integral surface of the equation $(x-y)y^2p + (y-x)x^2q = (x^2 + y^2)z$ which passes through the curve $xz = a^3, y = 0$. 5
Or

Solve :

$$\sqrt{p} + \sqrt{q} = 1$$

2. (a) Write the Jacobi's subsidiary equations. 2
(b) Find the complete integral of any one of the following : 4

(i) $(p^2 + q^2)y = qz$

(ii) $pxy + pq + qy = yz$

(iii) $p = (z + qy)^2$

- (c) Find the complete integral of $p_3x_3(p_1 + p_2) + x_1 + x_2 = 0$ 6
Or

Solve the boundary value problem $\frac{\partial u}{\partial x} - 2\frac{\partial u}{\partial y} = u$ with $u(x, 0) = 6e^{-3x}$ by the method of separation of variables.

3. (a) Write the Laplace equation. 1
(b) Classify the following equations :

(i) $(1-x^2)\frac{\partial^2 z}{\partial x^2} - 2xy\frac{\partial^2 z}{\partial x\partial y} + (1-y^2)\frac{\partial^2 z}{\partial y^2}$

$$+ 2x\frac{\partial z}{\partial x} + 6x^2y\frac{\partial z}{\partial y} - 6z = 0 \quad 2$$

(ii) $u_{xx} + u_{yy} + u_{zz} + u_{yz} + u_{zy} = 0 \quad 2$

(c) Reduce the equation

$$y(x+y)(r-s) - xp - yq - z = 0$$

to canonical form.

7

Or

Derive the one-dimensional wave equation.

4. (a) Fill in the blank :

The PDE in case of vibrating string problem is formulated from the law of _____.

1

(b) Write one-dimensional heat equation.

1

(c) Solve

$$\frac{\partial^2 u}{\partial x^2} - 2 \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} = 0$$

using the method of separation of variables.

6

Or

Find the solution of $\frac{\partial^2 y}{\partial t^2} = c^2 \frac{\partial^2 y}{\partial x^2}$ such

that $y = p_0 \cos pt$ where p_0 is constant when $x = l$ and $y = 0$ when $x = 0$.

5. (a) Give an example of a linear system of ordinary differential equation with variable coefficient.

1

(4)

- (b) Transform the linear differential equation $\frac{d^3x}{dt^3} + 2\frac{d^2x}{dt^2} - \frac{dx}{dt} - 2x = e^{3t}$ into system of first order differential equation. 2
- (c) Prove that $x = 2e^t, y = -3e^{2t}$ is the solution of $\frac{dx}{dt} = 5x + 2y, \frac{dy}{dt} = 3x + 4y$. 2
- (d) Describe the method of successive approximation. 4

Or

Find first two approximations of the function that approximate the exact solution of the equation $\frac{dy}{dx} = x + y, y(0) = 1$.

- (e) Find the general solution of the system :

$$\frac{dx}{dt} = x + 2y, \frac{dy}{dt} = 3x + 2y \quad 6$$

Or

Using operator method, find the general solution of

$$\frac{dx}{dt} + \frac{dy}{dt} - 2x - 4y = e^t, \frac{dx}{dt} + \frac{dy}{dt} - y = e^{4t}$$
