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1 SEM TDC MTMH (CBCS) C 1

2021

(Held in January/February, 2022)

MATHEMATICS

(Core)

Paper : C-1

(Calculus)

Full Marks : 60

Pass Marks : 24

Time : 3 hours

The figures in the margin indicate full marks for the questions

1. (a) Write the value of $\frac{d}{dx}(\cosh x)$. 1
- (b) Inverse hyperbolic sine is symmetric about a line. Write that line. 1
- (c) Write the value of y_n , if $y = \cos(4x+3)$. 1
- (d) Define point of inflection. 1
- (e) Find $\frac{d}{dx}(\tanh \sqrt{1+x^2})$. 2
- (f) Show that $\sinh x$ is an increasing function of x . 2

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(Turn Over)

(g) Show that $y = x^2$ is concave up on $(-\infty, \infty)$. 2

(h) Show that $\operatorname{cosech}^{-1} x = \sinh^{-1} \frac{1}{x}$. 3

Or

Find the asymptotes of

$$x^3 + 2x^2y - xy^2 - 2y^3 + 3xy + 3y^2 + x + 1 = 0$$

(i) Find y_n , if $y = \sin^3 x$. 3

Or

Find y_n , if $y = x^3 \sin x$.

(j) Evaluate (any one) : 4

(i) $\lim_{x \rightarrow 0} \frac{e^x - e^{\sin x}}{x - \sin x}$

(ii) $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\tan 5x}{\tan x}$

2. (a) Find $\int \tan^5 x \, dx$. 3

Or

Evaluate $\int_0^1 x^2 (1-x)^{\frac{3}{2}} \, dx$.

(b) Obtain the reduction formula for $\int \sin^n x \, dx$ 4

(c) Obtain the reduction formula for $\int x^n e^{ax} \, dx$ 4

(3)

Or

Find the volume of the solid generated by revolving the region between the parabola $x = y^2 + 1$ and the line $x = 3$ about the line $x = 3$.

- (d) Find the volume of the solid generated by revolving the region bounded by the curves and lines $y = x$, $y = -\frac{x}{2}$, $x = 2$ about the y -axis. 4
3. (a) Write the equation $x^2 + y^2 = 1$ in parametric form. 1
- (b) A function $y = f(x)$ is defined on $[a, b]$. Write the domain of the function after given a natural parametrization
 $x = t, y = f(t)$ 1
- (c) Write the parametric formula for $\frac{d^2y}{dx^2}$. 1
- (d) Write the equivalent Cartesian equation of the polar equation $r \cos \theta = 2$. 1
- (e) Find the eccentricity of the ellipse $2x^2 + y^2 = 2$. 2
- (f) Find the polar equation of $xy = 1$. 2
- (g) Find the Cartesian equation from the parametric equation
 $x = 4 \cot t, y = 2 \sin t, 0 \leq t \leq 2\pi$ 3

- (h) Find a parametrization for the curve having the lower half of the parabola $x - 1 = y^2$.

4

Or

Find an equation for the line tangent to the curve $x = 2 \cos t$, $y = 2 \sin t$ at the point $t = \frac{\pi}{4}$.

4. (a) Define limit of a vector valued function.

1

- (b) Let the position of a moving particle is given by

$$\vec{r}(t) = (\sec t)\hat{i} + (\tan t)\hat{j} + \frac{t^3}{3}\hat{k}$$

Find the acceleration at any time t .

2

- (c) Evaluate the integral

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} [(\sin t)\hat{i} + (1 + \cos t)\hat{j} + (\sec^2 t)\hat{k}] dt$$

3

- (d) Write the value of $[\vec{a} \ \vec{b} \ \vec{a}]$.

1

- (e) Let $\vec{U}(t)$ and $\vec{V}(t)$ are differentiable vector function of t . Show that

$$\frac{d}{dt}(\vec{U} \cdot \vec{V}) = \vec{U}' \cdot \vec{V} + \vec{U} \cdot \vec{V}'$$

3

Or

Find the normal component of acceleration of a moving particle.
